

Convexities : combinatorial parameters and interval functions

Pierre Duchet

Centre National de la Recherche Scientifique, Paris

Abstract

Convexity constitutes a huge domain of investigation, which reveals of theoretical major importance when we wish to understand the combinatorial properties of geometrical objects. But, at the present stage of its development, this approach resembles more a wild path through the bushes than a broad highway in calm weather. To clean the way, a more or less systematic use of the so-called “Interval Function”, seems powerful.

To argue this point of view, we shall discuss in this talk three problems that involves “combinatorial parameters” (i.e. mainly: Helly (h), Radon (r) and partition (p_k) numbers, Caratheodory (c) and exchange (e) numbers). For each problem, some typical results (most of them being new) illustrate different uses of “interval functions” (or, equivalently, of 2-ary operators).

A) Can we modelize the combinatorial behavior of a general abstract convexity structure by a graph geodetic convexity??

– For h and r a satisfactory answer has been obtained, while for p_k (partition problem) or c only one-way inequalities are known.

B) How to determine the combinatorial parameters of a graph geodetic convexity??

– We establish that “ c is at most e ”.

– We solve the case of the Radon number of a chordal graph.

C) How to obtain general inequalities involving combinatorial parameters of abstract convexity structures??

– We present, in case $c = 2$, the best possible inequality between h , r and c .

What is a “good” combinatorial parameter?? How to develop a theory for 2-ary functions?? Are k -ary functions? interesting?? Those concluding questions will be left wide open.